

November 1997

Screening Currents in Affine Current Algebra

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Abstract

In this paper screening currents of the second kind are considered. They are constructed in any affine current algebra for directions corresponding to simple roots with multiplicity one in a decomposition of the highest root on a set of simple roots. These expressions are precisely of the form previously conjectured to be valid for all directions in general affine current algebras. However, by working out explicitly the screening currents in the case of $SO(5)$ based on the Lie algebra B_2 , it is demonstrated that much more complicated structures appear in the general case. In the distinguished representation of affine $OSp(2|2)$ current superalgebra, the screening current of the second kind in the bosonic direction is also provided.

PACS: 11.25.Hf

Keywords: Conformal field theory; Affine current algebra; Free field realizations

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1 Introduction

Since the work by Wakimoto [1] on free field realizations of affine $SL(2)$ current algebra much effort has been made in obtaining similar constructions in the general case, a problem in principle solved by Feigin and Frenkel [2]. Recently two independent methods have led to general and explicit solutions [3, 4, 5]. The method used by Petersen, Yu and the present author [4, 5] gives particularly simple and compact free field realizations and is amenable of generalizations to affine current superalgebras [6].

Free field realizations enable one in principle to build integral representations for correlators in conformal field theory [7, 8, 9, 10, 11, 12, 13]. In a recent series of papers Petersen, Yu and the present author have carried out such a study for conformal field theory based on affine $SL(2)$ current algebra [12]. It turns out that screening operators of both the first and the second kinds are crucial for being able to treat the general case of degenerate representations [14] and in particular admissible representations [15]. In that connection it is also necessary to be able to handle fractional powers of free fields. Well defined rules for that have been established also in [12]. A particular interest in these techniques is due to their close relationship with 2D quantum gravity and string theory [16, 17].

In order to generalize our work on affine $SL(2)$ current algebra to higher groups and supergroups, one needs not only free field realizations of the affine currents but also of the screening currents and the primary fields. These are also provided in [4, 5, 6], though in the general case not for screening currents of the second kind. An expression, originally written down by Ito [18], was proposed as a candidate for screening currents of the second kind in the case of arbitrary affine current algebra, and a proof was provided in the case of $SL(r+1)$.

In this paper we shall prove that the proposal is valid in all affine current algebras for directions corresponding to simple roots with multiplicity *one* in a decomposition of the highest root on a set of simple roots. This includes all directions in $SL(r+1)$, and indeed the proof we present is identical to the one employed in [4, 5] for $SL(r+1)$. The main result in this paper is the explicit construction of the screening current of the second kind in the direction α_2 of multiplicity *two* in affine $SO(5)$ current algebra, based on the Lie algebra B_2 . The expression clearly demonstrates that much more complicated structures than previously anticipated appear in the general case. The construction is not unique in the form we present it. Rather, it involves an infinite summation over a variable which is only restricted to take on integer-spaced values and may thus be written as $n \in (\mathbb{Z} + a)$, where a is a free parameter. This does not come as a surprise since our techniques for handling fractional powers of free ghost fields allow for the introduction of adjustable monodromy parameters [12]. The observed non-uniqueness is then an indication that our result for the screening current is a fractional (and asymptotic) expansion of some expression. Our result diverges as $n!$ so it can at most be Borel summable.

The screening current of the second kind for the bosonic direction in the distinguished representation of $OSp(2|2)$ is also provided. This result does not differ substantially from the result for the screening current of the second kind in $OSp(1|2)$ [19], hitherto the only known screening current of the second kind in affine current superalgebras. Generalizations to higher groups and supergroups are currently being investigated.

The remaining part of this paper is organized as follows. Section 2 serves to fix notation and reviews some of the main results in [5]. In Section 3 we present a proof of the result in directions of simple roots with multiplicity one. The expressions for the screening currents of the second kind in the direction α_2 in affine $SO(5)$ current algebra and the bosonic direction in affine $OSp(2|2)$ current superalgebra are then given and proofs are outlined. Section 4 contains concluding remarks.

2 Notation

Let \mathfrak{g} be a simple Lie algebra of $\dim \mathfrak{g} = d$ and $\text{rank } \mathfrak{g} = r$. \mathfrak{h} is a Cartan subalgebra of \mathfrak{g} . The set of (positive) roots is denoted (Δ_+) Δ , and we write $\alpha > 0$ if $\alpha \in \Delta_+$. The simple roots are $\{\alpha_i\}_{i=1,\dots,r}$. θ is the highest root, while $\alpha^\vee = 2\alpha/\alpha^2$ is the root dual to α . Using the triangular decomposition

$$\mathfrak{g} = \mathfrak{g}_- \oplus \mathfrak{h} \oplus \mathfrak{g}_+ \quad (1)$$

the raising and lowering operators are denoted $e_\alpha \in \mathfrak{g}_+$ and $f_\alpha \in \mathfrak{g}_-$ respectively with $\alpha \in \Delta_+$, and $h_i \in \mathfrak{h}$ are the Cartan operators. We let j_a denote an arbitrary Lie algebra element. In the Cartan-Weyl basis we have

$$[h_i, e_\alpha] = (\alpha_i^\vee, \alpha) e_\alpha \quad , \quad [h_i, f_\alpha] = -(\alpha_i^\vee, \alpha) f_\alpha \quad (2)$$

and

$$[e_\alpha, f_\alpha] = h_\alpha = G^{ij}(\alpha_i^\vee, \alpha^\vee) h_j \quad (3)$$

where the metric G_{ij} is related to the Cartan matrix A_{ij} as

$$A_{ij} = \alpha_i^\vee \cdot \alpha_j = (\alpha_i^\vee, \alpha_j) = G_{ij} \alpha_j^2 / 2 \quad (4)$$

while the Cartan-Killing form κ_{ab} is

$$\kappa_{\alpha, -\beta} = \frac{2}{\alpha^2} \delta_{\alpha, \beta} \quad , \quad \kappa_{ij} = G_{ij} \quad (5)$$

The Weyl vector $\rho = \frac{1}{2} \sum_{\alpha > 0} \alpha$ satisfies $\rho \cdot \alpha_i^\vee = 1$. We use the convention $f_{-\alpha, -\beta}^{-\gamma} = -f_{\alpha, \beta}^\gamma$ so the standard symmetries of the structure coefficients may be summarized as

$$\begin{aligned} f_{-\alpha, -\beta}^{-\gamma} &= -f_{\alpha, \beta}^\gamma \quad , \quad f_{\beta, \alpha}^\gamma = -f_{\alpha, \beta}^\gamma \\ \frac{f_{\alpha, \beta}^{\alpha+\beta}}{(\alpha+\beta)^2} &= \frac{f_{\beta, -(\alpha+\beta)}^{-\alpha}}{\alpha^2} = \frac{f_{-(\alpha+\beta), \alpha}^{-\beta}}{\beta^2} \end{aligned} \quad (6)$$

The Dynkin labels Λ_k of the weight Λ are defined by

$$\Lambda = \Lambda_k \Lambda^k \quad , \quad \Lambda_k = (\alpha_k^\vee, \Lambda) \quad (7)$$

where $\{\Lambda^k\}_{k=1,\dots,r}$ is the set of fundamental weights satisfying

$$(\alpha_i^\vee, \Lambda^k) = \delta_i^k \quad (8)$$

Elements in \mathfrak{g}_+ or vectors in representation spaces (see below) are parametrized using “triangular coordinates” denoted by x^α , one for each positive root, thus we write general Lie algebra elements in \mathfrak{g}_+ as

$$g_+(x) = x^\alpha e_\alpha \in \mathfrak{g}_+ \quad (9)$$

and the corresponding group elements $G_+(x)$ as

$$G_+(x) = e^{g_+(x)} \quad (10)$$

The matrix representation $C(x)$ of $g_+(x)$ in the adjoint representation is defined by

$$C_a^b(x) = C(x)_a^b = (x^\beta C_\beta)_a^b = -x^\beta f_{\beta a}^b \quad (11)$$

and may be block decomposed as

$$C = \begin{pmatrix} C_+^+ & 0 & 0 \\ C_0^+ & 0 & 0 \\ C_-^+ & C_-^0 & C_-^- \end{pmatrix} \quad (12)$$

C_+^+ etc are matrices themselves. In C_+^+ both row and column indices are positive roots, in C_-^0 the row index is a negative root and the column index is a Cartan algebra index, etc. Note that $C_\alpha^\beta(x)$ vanishes unless $\alpha < \beta$, corresponding to C_+^+ being upper triangular with zeros in the diagonal. Similarly, C_-^- is lower triangular. We will understand “properly” repeated root indices as in (11) to be summed over the positive roots.

For the associated affine Lie algebra, the operator product expansion, OPE, of the associated currents is

$$J_a(z)J_b(w) = \frac{\kappa_{ab}k}{(z-w)^2} + \frac{f_{ab}^c J_c(w)}{z-w} \quad (13)$$

where regular terms have been omitted. k is the central extension and $k^\vee = 2k/\theta^2$ is the level. The Sugawara energy momentum tensor is

$$T(z) = \frac{1}{2t} \kappa^{ab} : J_a J_b : (z) \quad (14)$$

where we have introduced the parameter

$$t = \frac{\theta^2}{2} (k^\vee + h^\vee) \quad (15)$$

and where h^\vee is the dual Coxeter number. This tensor has central charge

$$c = \frac{k^\vee d}{k^\vee + h^\vee} \quad (16)$$

The standard free field construction [1, 2, 20, 21, 22, 18, 23, 24, 25, 26] consists in introducing for every positive root $\alpha > 0$, a pair of free bosonic ghost fields $(\beta_\alpha, \gamma^\alpha)$ of conformal weights (1,0) satisfying the OPE

$$\beta_\alpha(z)\gamma^\beta(w) = \frac{\delta_\alpha^\beta}{z-w} \quad (17)$$

The corresponding energy-momentum tensor is

$$T_{\beta\gamma}(z) =: \partial\gamma^\alpha(z)\beta_\alpha(z) : \quad (18)$$

with central charge

$$c_{\beta\gamma} = d - r \quad (19)$$

For every Cartan index $i = 1, \dots, r$ one introduces a free scalar boson φ_i with contraction

$$\varphi_i(z)\varphi_j(w) = G_{ij} \ln(z - w) \quad (20)$$

The energy-momentum tensor

$$T_\varphi(z) = \frac{1}{2} : \partial\varphi(z) \cdot \partial\varphi(z) : - \frac{1}{\sqrt{t}} \rho \cdot \partial^2 \varphi(z) \quad (21)$$

has central charge

$$c_\varphi = r - \frac{h^\vee d}{k^\vee + h^\vee} \quad (22)$$

This follows from Freudenthal-de Vries strange formula $\rho^2 = h^\vee \theta^2 d / 24$. The combination $T_\varphi + T_{\beta\gamma}$ is the free field realization of the Sugawara energy-momentum tensor.

The vertex operator

$$\begin{aligned} V_\Lambda(z) &= : e^{\frac{1}{\sqrt{t}} \Lambda \cdot \varphi(z)} : \\ \Lambda \cdot \varphi(z) &= \Lambda_i G^{ij} \varphi_j(z) \end{aligned} \quad (23)$$

has conformal weight

$$\Delta(V_\Lambda) = \frac{1}{2t} (\Lambda, \Lambda + 2\rho) \quad (24)$$

It is also affine primary corresponding to highest weight Λ . In [5], the explicit general construction is provided of the full multiplet of primary fields, parametrized by the x^α coordinates. A similar and likewise general construction in the case of superalgebras is provided in [6].

2.1 Differential Operator Realization

The lowest weight vector in the (dual) representation space $\langle \Lambda |$ is introduced as

$$\langle \Lambda | f_\alpha = 0 \quad , \quad \langle \Lambda | h_i = \Lambda_i \langle \Lambda | \quad (25)$$

An arbitrary vector in this representation space is parametrized as

$$\langle \Lambda, x | = \langle \Lambda | G_+(x) \quad (26)$$

A differential operator realization $\tilde{J}_a(x, \partial, \Lambda)$ of the simple Lie algebra \mathfrak{g} may then be defined by

$$\langle \Lambda, x | j_a = \tilde{J}_a(x, \partial, \Lambda) \langle \Lambda, x | \quad (27)$$

with $\partial_\alpha = \partial_{x^\alpha}$ denoting partial derivative wrt x^α . It is obvious that these operators satisfy the Lie algebra commutation relations. From the Gauss decomposition of $\langle \Lambda | G_+(x) e^{tj_a}$, for t small,

$$\begin{aligned}
\langle \Lambda | G_+(x) \exp(te_\alpha) &= \langle \Lambda | \exp \left(x^\gamma e_\gamma + tV_\alpha^\beta(x) e_\beta + \mathcal{O}(t^2) \right) \\
&= \langle \Lambda | \exp \left(tV_\alpha^\beta(x) \partial_\beta + \mathcal{O}(t^2) \right) G_+(x) \\
\langle \Lambda | G_+(x) \exp(th_i) &= \langle \Lambda | \exp(th_i) \exp \left(x^\gamma e_\gamma + tV_i^\beta(x) e_\beta + \mathcal{O}(t^2) \right) \\
&= \langle \Lambda | \exp \left(t \left(V_i^\beta(x) \partial_\beta + \Lambda_i \right) + \mathcal{O}(t^2) \right) G_+(x) \\
\langle \Lambda | G_+(x) \exp(tf_\alpha) &= \langle \Lambda | \exp \left(tQ_{-\alpha}^{-\beta}(x) f_\beta + \mathcal{O}(t^2) \right) \exp \left(tP_{-\alpha}^j(x) h_j + \mathcal{O}(t^2) \right) \\
&\quad \cdot \exp \left(x^\gamma e_\gamma + tV_{-\alpha}^\beta(x) e_\beta + \mathcal{O}(t^2) \right) \\
&= \langle \Lambda | \exp \left(t \left(P_{-\alpha}^j(x) \Lambda_j + V_{-\alpha}^\beta(x) \partial_\beta \right) + \mathcal{O}(t^2) \right) G_+(x) \quad (28)
\end{aligned}$$

it follows that the differential operator realization is of the form

$$\begin{aligned}
\tilde{E}_\alpha(x, \partial) &= V_\alpha^\beta(x) \partial_\beta \\
\tilde{H}_i(x, \partial, \Lambda) &= V_i^\beta(x) \partial_\beta + \Lambda_i \\
\tilde{F}_\alpha(x, \partial, \Lambda) &= V_{-\alpha}^\beta(x) \partial_\beta + P_{-\alpha}^j(x) \Lambda_j \quad (29)
\end{aligned}$$

and one finds [4, 5]

$$\begin{aligned}
V_\alpha^\beta(x) &= [B(C(x))]_\alpha^\beta \\
V_i^\beta(x) &= -[C(x)]_i^\beta \\
V_{-\alpha}^\beta(x) &= \left[e^{-C(x)} \right]_{-\alpha}^\gamma [B(-C(x))]_\gamma^\beta \\
P_{-\alpha}^j(x) &= \left[e^{-C(x)} \right]_{-\alpha}^j \\
Q_{-\alpha}^{-\beta}(x) &= \left[e^{-C(x)} \right]_{-\alpha}^{-\beta} \quad (30)
\end{aligned}$$

B is the generating function for the Bernoulli numbers

$$\begin{aligned}
B(u) &= \frac{u}{e^u - 1} = \sum_{n \geq 0} \frac{B_n}{n!} u^n \\
B^{-1}(u) &= \frac{e^u - 1}{u} = \sum_{n \geq 0} \frac{1}{(n+1)!} u^n \quad (31)
\end{aligned}$$

The matrix functions (30) are defined in terms of universal power series expansions, valid for any Lie algebra, but ones that truncate giving rise to finite polynomials of which the explicit forms depend on the Lie algebra in question.

The differential screening operators are defined by

$$\begin{aligned}
\exp\{-te_\alpha\} G_+(x) &= \exp\{tS_\alpha(x, \partial) + \mathcal{O}(t^2)\} G_+(x) \\
S_\alpha(x, \partial) &= S_\alpha^\beta(x) \partial_\beta \quad (32)
\end{aligned}$$

and are seen to satisfy

$$S_\alpha(x, \partial) = \tilde{E}_\alpha(-x, -\partial) \quad (33)$$

so that

$$S_\alpha^\beta(x) = -[B(-C(x))]^\beta_\alpha \quad (34)$$

The screening currents of the first kind are constructed using these polynomials for simple roots as building blocks, see below.

2.2 Free Field Realization

The free field realization is obtained from the differential operator realization $\{\tilde{J}_a\}$ by the substitution [2, 20, 21, 23, 25, 26]

$$\partial_\alpha \rightarrow \beta_\alpha(z) \quad , \quad x^\alpha \rightarrow \gamma^\alpha(z) \quad , \quad \Lambda_i \rightarrow \sqrt{t}\partial\varphi_i(z) \quad (35)$$

and a subsequent addition of a normal ordering contribution or anomalous term to the lowering part, giving rise to the following form of the free field realization

$$\begin{aligned} E_\alpha(z) &= :V_\alpha^\beta(\gamma(z))\beta_\beta(z): \\ H_i(z) &= :V_i^\beta(\gamma(z))\beta_\beta(z): + \sqrt{t}\partial\varphi_i(z) \\ F_\alpha(z) &= :V_{-\alpha}^\beta(\gamma(z))\beta_\beta(z): + \sqrt{t}\partial\varphi_j(z)P_{-\alpha}^j(\gamma(z)) + \partial\gamma^\beta(z)F_{\alpha\beta}(\gamma(z)) \\ \Delta(J_a) &= 1 \end{aligned} \quad (36)$$

In [5] (see also [4, 3] and [6]) the explicit form of $F_{\alpha\beta}$ is found to be

$$F_{\alpha\beta}(\gamma) = \frac{2k}{\alpha^2} \left((V_+^+(\gamma))^{-1} \right)_\beta^\alpha + \left((V_+^+(\gamma))^{-1} \right)_\beta^\mu \partial_\sigma V_\mu^\gamma(\gamma) \partial_\gamma V_{-\alpha}^\sigma(\gamma) \quad (37)$$

where

$$\begin{aligned} (V_+^+(\gamma))^{-1} &= B(C_+^+(\gamma))^{-1} \\ &= \sum_{n \geq 0} \frac{1}{(n+1)!} (C_+^+(\gamma))^n \end{aligned} \quad (38)$$

2.3 Screening Currents of the First Kind

A screening current is conformally primary of weight 1 and has the property that the singular part of the OPE with an affine current is a total derivative. These properties ensure that integrated screening currents (screening charges) may be inserted into correlators without altering the conformal or affine Ward identities. This in turn makes them very useful in construction of correlators, see e.g. [7, 10, 24, 12, 4]. The best known screening currents [2, 21, 18, 25, 26, 3, 4, 5] are the following denoted screening currents of the first kind, one for each simple root

$$\begin{aligned} s_j(w) &= :S_{\alpha_j}^\alpha(\gamma(w))\beta_\alpha(w): e^{-\frac{1}{\sqrt{t}}\alpha_j \cdot \varphi(w)} : \\ \alpha_j \cdot \varphi(w) &= \frac{\alpha_j^2}{2} \varphi_j(w) \end{aligned} \quad (39)$$

satisfying

$$\begin{aligned}
E_\alpha(z)s_j(w) &= 0 \\
H_i(z)s_j(w) &= 0 \\
F_\alpha(z)s_j(w) &= \frac{\partial}{\partial w} \left(\frac{-2t/\alpha_j^2}{z-w} Q_{-\alpha}^{-\alpha_j}(\gamma(w)) : e^{-\frac{1}{\sqrt{t}}\alpha_j \cdot \varphi(w)} : \right) \\
T(z)s_j(w) &= \frac{\partial}{\partial w} \left(\frac{1}{z-w} s_j(w) \right)
\end{aligned} \tag{40}$$

We shall use the terminology that $s_j(z)$ is the screening current of the first kind in the direction α_j .

3 Screening Currents of the Second Kind

In [27] Bershadsky and Ooguri found a second screening current in the case of $SL(2)$

$$\tilde{s}(w) = (-\beta(w))^{-(k^\vee + h^\vee)} : e^{\sqrt{t}\varphi_1(w)} : \tag{41}$$

Since it involves a generically non-integer power of the free ghost field β , discussions on its interpretation remained only partly successful. However, in [12, 4] by Petersen, Yu and the present author it is demonstrated how techniques of fractional calculus provide a solution. As a result we were able to render the free field realization applicable of producing integral representations of N -point chiral blocks for degenerate representations and in particular for admissible representations.

The problem of extending the construction of Bershadsky and Ooguri to higher groups is discussed in [4, 5] where a general proposal is studied, originally written down by Ito [18]. In the case of $SL(r+1)$ we there presented a proof of this proposal but for general groups it has remained a conjecture. However, Proposition 1 below demonstrates that in general the validity of that proposal is restricted to certain directions. We shall use the following notation:

$$\theta = \sum_{i=1}^r a^i \alpha_i \tag{42}$$

is the decomposition of the highest root θ on the space of simple roots, while

$$\begin{aligned}
\tilde{s}_j(w) &= \tilde{S}_j(w) : e^{\sqrt{t}\varphi_j(w)} : \\
J_a(z)\tilde{s}_j(w) &= \frac{\partial}{\partial w} \left(\frac{1}{z-w} R_{a,\alpha_j}(w) : e^{\sqrt{t}\varphi_j(w)} : \right)
\end{aligned} \tag{43}$$

is the contraction between the affine current $J_a(z)$ and the screening current of the second kind $\tilde{s}_j(w)$ in the direction α_j . $\tilde{S}_j(w)$ and $R_{a,\alpha_j}(w)$ are assumed to be functionals of the ghost fields and derivatives thereof.

Proposition 1

In the direction of the simple root α_j of multiplicity one, $a^j = 1$, the screening current of the second kind is given by

$$\tilde{s}_j(w) =: \left(S_{\alpha_j}^\sigma(\gamma(w)) \beta_\sigma(w) \right)^{-2t/\alpha_j^2} :: e^{\sqrt{t}\varphi_j(w)} : \tag{44}$$

and it produces

$$\begin{aligned}
R_{\alpha, \alpha_j} &= R_{i, \alpha_j} = 0 \\
R_{-\alpha, \alpha_j} &= -\frac{2t}{\alpha_j^2} : Q_{-\alpha}^{-\alpha_j}(\gamma) (S_{\alpha_j}^\sigma(\gamma) \beta_\sigma)^{-2t/\alpha_j^2-1} : \\
\Delta(\tilde{s}_j) &= 1
\end{aligned} \tag{45}$$

Proof

First one computes all possible (multiple) contractions between a generator (J_a or T) and the screening current \tilde{s}_j . Comparisons with (45) will then yield a set of relations among the polynomials V , P , Q and S . These are then proven using the various classical and quantum polynomial identities which follow from the fact that indeed (29) and (36) constitute a differential operator realization and a free field realization, respectively. [4, 5, 6] may be consulted for details and lists of polynomial identities. In this final part of the proof, the essential point is that for $a^j = 1$ certain "contracted sequences" of polynomials like $S_{\alpha_j}^\gamma \partial_\gamma S_{\alpha_j}^\sigma$ and $S_{\alpha_j}^{\gamma_1} S_{\alpha_j}^{\gamma_2} S_{\alpha_j}^{\gamma_3} \partial_{\gamma_1} \partial_{\gamma_2} \partial_{\gamma_3} V_{-\alpha}^\beta$ all vanish; that is when α_j net appears more than once in the lower indices. This simple rule follows immediately from the expressions (30), (34) and (37) and the definition of the matrix C (11) and (12). In general $S_{\alpha_j}^\gamma \partial_\gamma S_{\alpha_j}^\sigma \partial_\sigma V_{-\alpha}^\beta$ will not vanish. It is similarly obvious that contracted sequences vanish if the sum of roots in the upper indices is less than the net sum of roots in the lower indices.

□

Note that the 3rd order pole in the OPE $T(z)\tilde{s}_j(w)$ is proportional to $S_{\alpha_j}^\sigma \partial_\sigma S_{\alpha_j}^\beta$ which is generically non-vanishing for $a^j > 1$, showing the limited validity of the expression (44).

3.1 Case of $SO(5)$

Here we shall consider the case $SO(5)$ where the affine current algebra is based on the simple Lie algebra B_2 . This Lie algebra has rank $r = 2$, 4 positive roots and dimension 10, while the dual Coxeter number is $h^\vee = 3$. We shall use the notation where $\alpha_{11} = \alpha_1 + \alpha_2$ and

$$\theta = \alpha_1 + 2\alpha_2 \tag{46}$$

denote the two non-simple positive roots. The normalization of the root system is such that the highest root θ has length squared $\theta^2 = \alpha_1^2 = 2\alpha_2^2 = 2\alpha_{11}^2 = 2$. The Cartan matrix and the Cartan-Killing form are given by

$$\begin{aligned}
A_{11} = 2 & \quad , & A_{12} = -1 & \quad , & A_{21} = -2 & \quad , & A_{22} = 2 \\
G_{11} = 2 & \quad , & G_{12} = -2 & \quad , & G_{21} = -2 & \quad , & G_{22} = 4 \\
\kappa_{\alpha, -\alpha} &= 2/\alpha^2
\end{aligned} \tag{47}$$

The remaining structure coefficients are (up to the symmetries (2) and (6))

$$\begin{aligned}
f_{\alpha_1, \alpha_2}^{\alpha_{11}} &= 1 & , & & f_{\alpha_2, \alpha_{11}}^\theta &= 2 \\
f_{1, \alpha_{11}}^{\alpha_{11}} &= 1 & , & & f_{2, \alpha_{11}}^{\alpha_{11}} &= 0
\end{aligned}$$

$$\begin{aligned}
f_{1,\theta}{}^\theta &= 0 & , & & f_{2,\theta}{}^\theta &= 2 \\
f_{\alpha_{11},-\alpha_{11}}{}^1 &= 2 & , & & f_{\alpha_{11},-\alpha_{11}}{}^2 &= 1 \\
f_{\theta,-\theta}{}^1 &= 1 & , & & f_{\theta,-\theta}{}^2 &= 1
\end{aligned} \tag{48}$$

The differential operator realization is worked out to be

$$\begin{aligned}
e_{\alpha_1}(x) &= \partial_1 - \frac{1}{2}x^2\partial_{11} - \frac{1}{6}x^2x^2\partial_\theta \\
e_{\alpha_2}(x) &= \partial_2 + \frac{1}{2}x^1\partial_{11} + \left(\frac{1}{6}x^1x^2 - x^{11}\right)\partial_\theta \\
e_{\alpha_{11}}(x) &= \partial_{11} + x^2\partial_\theta \\
e_\theta(x) &= \partial_\theta \\
h_1(x) &= -2x^1\partial_1 + x^2\partial_2 - x^{11}\partial_{11} + \Lambda_1 \\
h_2(x) &= 2x^1\partial_1 - 2x^2\partial_2 - 2x^\theta\partial_\theta + \Lambda_2 \\
f_{\alpha_1}(x) &= -x^1x^1\partial_1 + \left(\frac{1}{2}x^1x^2 - x^{11}\right)\partial_2 - \frac{1}{2}x^1\left(\frac{1}{2}x^1x^2 + x^{11}\right)\partial_{11} \\
&\quad + \frac{1}{3}x^1x^2x^{11}\partial_\theta + x^1\Lambda_1 \\
f_{\alpha_2}(x) &= 2\left(\frac{1}{2}x^1x^2 + x^{11}\right)\partial_1 - x^2x^2\partial_2 + \left(\frac{1}{3}x^1x^2x^2 - x^\theta\right)\partial_{11} \\
&\quad - x^2\left(\frac{1}{3}x^2x^{11} + x^\theta\right)\partial_\theta + x^2\Lambda_2 \\
f_{\alpha_{11}}(x) &= -2x^1\left(\frac{1}{2}x^1x^2 + x^{11}\right)\partial_1 + \left(\frac{2}{3}x^1x^2x^2 - x^2x^{11} + x^\theta\right)\partial_2 \\
&\quad + \left(-\frac{5}{12}x^1x^1x^2x^2 - \frac{1}{2}x^1x^2x^{11} + \frac{1}{2}x^1x^\theta - x^{11}x^{11}\right)\partial_{11} \\
&\quad + \left(\frac{1}{36}x^1x^1x^2x^2x^2 + \frac{1}{2}x^1x^2x^2x^{11} + \frac{1}{6}x^1x^2x^\theta - x^{11}x^\theta\right)\partial_\theta \\
&\quad + 2\left(\frac{1}{2}x^1x^2 + x^{11}\right)\Lambda_1 - \left(\frac{1}{2}x^1x^2 - x^{11}\right)\Lambda_2 \\
f_\theta(x) &= \left(\frac{1}{4}x^1x^1x^2x^2 + x^1x^2x^{11} + x^{11}x^{11}\right)\partial_1 - x^2\left(\frac{1}{6}x^1x^2x^2 + x^\theta\right)\partial_2 \\
&\quad + \left(\frac{1}{8}x^1x^1x^2x^2x^2 + \frac{1}{3}x^1x^2x^2x^{11} + \frac{1}{2}x^2x^{11}x^{11} - x^{11}x^\theta\right)\partial_{11} \\
&\quad - \left(\frac{1}{72}x^1x^1x^2x^2x^2x^2 + \frac{1}{6}x^1x^2x^2x^2x^{11} + \frac{1}{6}x^2x^2x^{11}x^{11} + x^\theta x^\theta\right)\partial_\theta \\
&\quad + \left(-\frac{1}{3}x^1x^2x^2 - x^2x^{11} + x^\theta\right)\Lambda_1 + \left(\frac{1}{6}x^1x^2x^2 + x^\theta\right)\Lambda_2
\end{aligned} \tag{49}$$

Here we have introduced the simplifying notation $x^{11} = x^{\alpha_{11}}$, $\partial_1 = \partial_{\alpha_1}$ etc. In the following we shall also need the analogous abbreviations $\gamma^2(z) = \gamma^{\alpha_2}(z)$, $\beta_{11}(z) = \beta_{\alpha_{11}}(z)$ etc. Furthermore, the differential screening operators are

$$\begin{aligned}
S_{\alpha_1}^\beta(x) &= -\partial_1 - \frac{1}{2}x^2\partial_{11} + \frac{1}{6}x^2x^2\partial_\theta \\
S_{\alpha_2}^\beta(x) &= -\partial_2 + \frac{1}{2}x^1\partial_{11} - \left(\frac{1}{6}x^1x^2 + x^{11}\right)\partial_\theta
\end{aligned} \tag{50}$$

The (generalized) Wakimoto free field realization of the associated affine current algebra becomes

$$\begin{aligned}
E_{\alpha_1} &= \beta_1 - \frac{1}{2}\gamma^2\beta_{11} - \frac{1}{6}\gamma^2\gamma^2\beta_\theta \\
E_{\alpha_2} &= \beta_2 + \frac{1}{2}\gamma^1\beta_{11} + \left(\frac{1}{6}\gamma^1\gamma^2 - \gamma^{11}\right)\beta_\theta \\
E_{\alpha_{11}} &= \beta_{11} + \gamma^2\beta_\theta \\
E_\theta &= \beta_\theta \\
H_1 &= -2 : \gamma^1\beta_1 : + : \gamma^2\beta_2 : - : \gamma^{11}\beta_{11} : + \sqrt{t}\partial\varphi_1 \\
H_2 &= 2 : \gamma^1\beta_1 : - 2 : \gamma^2\beta_2 : - 2 : \gamma^\theta\beta_\theta : + \sqrt{t}\partial\varphi_2 \\
F_{\alpha_1} &= - : \gamma^1\gamma^1\beta_1 : + : \left(\frac{1}{2}\gamma^1\gamma^2 - \gamma^{11}\right)\beta_2 : - \frac{1}{2} : \gamma^1\left(\frac{1}{2}\gamma^1\gamma^2 + \gamma^{11}\right)\beta_{11} : \\
&\quad + \frac{1}{3}\gamma^1\gamma^2\gamma^{11}\beta_\theta + \sqrt{t}\gamma^1\partial\varphi_1 + \left(k + \frac{1}{2}\right)\partial\gamma^1 \\
F_{\alpha_2} &= 2 : \left(\frac{1}{2}\gamma^1\gamma^2 + \gamma^{11}\right)\beta_1 : - : \gamma^2\gamma^2\beta_2 : + \left(\frac{1}{3}\gamma^1\gamma^2\gamma^2 - \gamma^\theta\right)\beta_{11} \\
&\quad - : \gamma^2\left(\frac{1}{3}\gamma^2\gamma^{11} + \gamma^\theta\right)\beta_\theta : + \sqrt{t}\gamma^2\partial\varphi_2 + 2(k+1)\partial\gamma^2 \\
F_{\alpha_{11}} &= -2 : \gamma^1\left(\frac{1}{2}\gamma^1\gamma^2 + \gamma^{11}\right)\beta_1 : + : \left(\frac{2}{3}\gamma^1\gamma^2\gamma^2 - \gamma^2\gamma^{11} + \gamma^\theta\right)\beta_2 : \\
&\quad + : \left(-\frac{5}{12}\gamma^1\gamma^1\gamma^2\gamma^2 - \frac{1}{2}\gamma^1\gamma^2\gamma^{11} + \frac{1}{2}\gamma^1\gamma^\theta - \gamma^{11}\gamma^{11}\right)\beta_{11} : \\
&\quad + : \left(\frac{1}{36}\gamma^1\gamma^1\gamma^2\gamma^2\gamma^2 + \frac{1}{2}\gamma^1\gamma^2\gamma^2\gamma^{11} + \frac{1}{6}\gamma^1\gamma^2\gamma^\theta - \gamma^{11}\gamma^\theta\right)\beta_\theta : \\
&\quad + 2\sqrt{t}\left(\frac{1}{2}\gamma^1\gamma^2 + \gamma^{11}\right)\partial\varphi_1 - \sqrt{t}\left(\frac{1}{2}\gamma^1\gamma^2 - \gamma^{11}\right)\partial\varphi_2 \\
&\quad + \left(k + \frac{2}{3}\right)\gamma^2\partial\gamma^1 - \left(k + \frac{11}{6}\right)\gamma^1\partial\gamma^2 + (2k+1)\partial\gamma^{11} \\
F_\theta &= : \left(\frac{1}{4}\gamma^1\gamma^1\gamma^2\gamma^2 + \gamma^1\gamma^2\gamma^{11} + \gamma^{11}\gamma^{11}\right)\beta_1 : - : \gamma^2\left(\frac{1}{6}\gamma^1\gamma^2\gamma^2 + \gamma^\theta\right)\beta_2 : \\
&\quad + : \left(\frac{1}{8}\gamma^1\gamma^1\gamma^2\gamma^2\gamma^2 + \frac{1}{3}\gamma^1\gamma^2\gamma^2\gamma^{11} + \frac{1}{2}\gamma^2\gamma^{11}\gamma^{11} - \gamma^{11}\gamma^\theta\right)\beta_{11} : \\
&\quad - : \left(\frac{1}{72}\gamma^1\gamma^1\gamma^2\gamma^2\gamma^2\gamma^2 + \frac{1}{6}\gamma^1\gamma^2\gamma^2\gamma^2\gamma^{11} + \frac{1}{6}\gamma^2\gamma^2\gamma^{11}\gamma^{11} + \gamma^\theta\gamma^\theta\right)\beta_\theta : \\
&\quad + \sqrt{t}\left(-\frac{1}{3}\gamma^1\gamma^2\gamma^2 - \gamma^2\gamma^{11} + \gamma^\theta\right)\partial\varphi_1 + \sqrt{t}\left(\frac{1}{6}\gamma^1\gamma^2\gamma^2 + \gamma^\theta\right)\partial\varphi_2 \\
&\quad - \frac{1}{3}\left(k + \frac{1}{2}\right)\gamma^2\gamma^2\partial\gamma^1 + \frac{1}{3}\left(k + \frac{5}{2}\right)\gamma^1\gamma^2\partial\gamma^2 + (k+2)\gamma^{11}\partial\gamma^2 \\
&\quad - (k+1)\gamma^2\partial\gamma^{11} + k\partial\gamma^\theta
\end{aligned} \tag{51}$$

For notational reasons we have left out the arguments which are the same for all the fields. The screening currents of the first kind are found to be

$$s_1(w) = \left(-\beta_1(w) - \frac{1}{2}\gamma^2(w)\beta_{11}(w) + \frac{1}{6}\gamma^2(w)\gamma^2(w)\beta_\theta(w)\right) : e^{-\varphi_1(w)/\sqrt{t}} :$$

$$\begin{aligned}
s_2(w) &= \left(-\beta_2(w) + \frac{1}{2}\gamma^1(w)\beta_{11}(w) - \left(\frac{1}{6}\gamma^1(w)\gamma^2(w) + \gamma^{11}(w) \right) \beta_\theta(w) \right) \\
&\quad \cdot : e^{-\varphi_2(w)/(2\sqrt{t})} :
\end{aligned} \tag{52}$$

Due to the decomposition (46), it follows from Proposition 1 that the screening current $\tilde{s}_1(w)$ is given by

$$\tilde{s}_1(w) = \left(-\beta_1(w) - \frac{1}{2}\gamma^2(w)\beta_{11}(w) + \frac{1}{6}\gamma^2(w)\gamma^2(w)\beta_\theta(w) \right)^{-t} : e^{\sqrt{t}\varphi_1(w)} : \tag{53}$$

Note that in this case the normal ordering of the $\beta\gamma$ part is not necessary. It may be checked explicitly that the OPE's with the generators $\{J_a(z)\}$ and $T(z)$ produce

$$\begin{aligned}
R_{-\alpha_1, \alpha_1} &= -t \left(-\beta_1 - \frac{1}{2}\gamma^2\beta_{11} + \frac{1}{6}\gamma^2\gamma^2\beta_\theta \right)^{-t-1} \\
R_{-\alpha_{11}, \alpha_1} &= -2t\gamma^2 \left(-\beta_1 - \frac{1}{2}\gamma^2\beta_{11} + \frac{1}{6}\gamma^2\gamma^2\beta_\theta \right)^{-t-1} \\
R_{-\theta, \alpha_1} &= t\gamma^2\gamma^2 \left(-\beta_1 - \frac{1}{2}\gamma^2\beta_{11} + \frac{1}{6}\gamma^2\gamma^2\beta_\theta \right)^{-t-1} \\
R_{-\alpha_2, \alpha_1} &= R_{\alpha, \alpha_1} = R_{i, \alpha_1} = 0 \\
\Delta(\tilde{s}_1) &= 1
\end{aligned} \tag{54}$$

These expressions comply with the general statement (45) since

$$\begin{aligned}
Q_{-\alpha_1}^{-\alpha_1}(x) &= 1 & , & & Q_{-\alpha_{11}}^{-\alpha_1}(x) &= 2x^2 \\
Q_{-\alpha_2}^{-\alpha_1}(x) &= 0 & , & & Q_{-\theta}^{-\alpha_1}(x) &= -x^2x^2
\end{aligned} \tag{55}$$

Proposition 2

The screening current $\tilde{s}_2(w)$ is given by

$$\begin{aligned}
\tilde{s}_2(w) &= \sum_n C_n : \left(-\frac{1}{3} \left(2\partial\gamma^1(w)\beta_\theta(w) - \gamma^1(w)\partial\beta_\theta(w) \right) \right)^n \\
&\quad \cdot \left(-\beta_2(w) + \frac{1}{2}\gamma^1(w)\beta_{11}(w) - \left(\gamma^{11}(w) + \frac{1}{6}\gamma^1(w)\gamma^2(w) \right) \beta_\theta(w) \right)^{-2t-2n} : \\
&\quad \cdot : e^{\sqrt{t}\varphi_2(w)} : \\
C_n &= \frac{1}{2^n n!} \frac{(-2t)!}{(-2t-2n)!}
\end{aligned} \tag{56}$$

and produces

$$\begin{aligned}
R_{-\alpha_2, \alpha_2} &= \sum_n C_n (-2t-2n) : \left(-\frac{1}{3} \left(2\partial\gamma^1\beta_\theta - \gamma^1\partial\beta_\theta \right) \right)^n \\
&\quad \cdot \left(-\beta_2 + \frac{1}{2}\gamma^1\beta_{11} - \left(\gamma^{11} + \frac{1}{6}\gamma^1\gamma^2 \right) \beta_\theta \right)^{-2t-2n-1} : \\
R_{-\alpha_{11}, \alpha_2} &= \sum_n C_n (2t+2n) : \gamma^1 \left(-\frac{1}{3} \left(2\partial\gamma^1\beta_\theta - \gamma^1\partial\beta_\theta \right) \right)^n
\end{aligned}$$

$$\begin{aligned}
& \cdot \left(-\beta_2 + \frac{1}{2}\gamma^1\beta_{11} - \left(\gamma^{11} + \frac{1}{6}\gamma^1\gamma^2 \right) \beta_\theta \right)^{-2t-2n-1} : \\
R_{-\theta, \alpha_2} &= \sum_n C_n n : \partial\gamma^1 \left(-\frac{1}{3} \left(2\partial\gamma^1\beta_\theta - \gamma^1\partial\beta_\theta \right) \right)^{n-1} \\
& \cdot \left(-\beta_2 + \frac{1}{2}\gamma^1\beta_{11} - \left(\gamma^{11} + \frac{1}{6}\gamma^1\gamma^2 \right) \beta_\theta \right)^{-2t-2n} : \\
& + \sum_n C_n (-2t-2n) : \left(\frac{1}{2}\gamma^1\gamma^2 + \gamma^{11} \right) \left(-\frac{1}{3} \left(2\partial\gamma^1\beta_\theta - \gamma^1\partial\beta_\theta \right) \right)^n \\
& \cdot \left(-\beta_2 + \frac{1}{2}\gamma^1\beta_{11} - \left(\gamma^{11} + \frac{1}{6}\gamma^1\gamma^2 \right) \beta_\theta \right)^{-2t-2n-1} : \\
R_{-\alpha_1, \alpha_2} &= R_{\alpha, \alpha_2} = R_{i, \alpha_2} = 0 \\
\Delta(\tilde{s}_2) &= 1
\end{aligned} \tag{57}$$

The summation over n is not restricted to be a summation over integers, but is given by $n \in (\mathbb{Z} + a)$ where a is a free parameter. For b non-integer, the factorial $b!$ is defined by the gamma-function $b! = \Gamma(b+1)$.

Proof

The strategy is straightforward, though very tedious. It amounts to compute all possible (multiple) contractions between a generator and the screening current for all generators and then reduce the expressions to obtain the total derivatives given above, (43) and (57). In the process we employ the recursion relation

$$(-2t-2n)(-2t-2n-1)C_n = 2(n+1)C_{n+1} \tag{58}$$

Note that this relation is valid also for n non-integer; that is for a non-integer.

□

The non-uniqueness of the expression (56) parameterized by a is believed to be an artifact of our representation of the result. In [12, 4] (see also [28]) it is discussed how fractional (and asymptotic) expansions like

$$(1+z)_{(a)}^u = \sum_{n \in \mathbb{Z}} \binom{u}{n+a} z^{n+a} \tag{59}$$

are relevant when computing contractions between ghost fields raised to non-integer powers. In that connection a plays the role of an adjustable monodromy parameter. We believe that a similar situation is encountered here and that our result for the screening current is merely a fractional expansion of some expression. Since our result diverges as $n!$ it can at most be an asymptotic (fractional) expansion, or a Borel summable expression.

3.2 Case of $OSp(2|2)$

Here we shall present the screening current of the second kind in the bosonic direction in the distinguished representation of affine $OSp(2|2)$ current superalgebra. The notation and results used here are taken from [6] which may be consulted for further details and general results in affine current superalgebras.

The Lie superalgebra $A(1,0)$, which is isomorphic to $osp(2|2)$, $C(2)$ and $sl(1|2) \simeq sl(2|1)$, has rank $r = 2$ and 3 positive roots, while the dual Coxeter number is $h^\vee = 1$ and $\dim(\mathfrak{g}_0) = \dim(\mathfrak{g}_1) = 4$. Here we choose the distinguished set of simple roots consisting of one even simple root α_1 and one odd simple root $\dot{\alpha}_2$. The remaining and non-simple root $\dot{\alpha} = \alpha_1 + \dot{\alpha}_2$ is then odd. The Weyl vector is $\rho = -\dot{\alpha}_2$, while the non-vanishing elements of the Cartan-Killing form are

$$\begin{aligned} G_{11} = 2 \quad , \quad G_{12} = G_{21} = -1 \quad , \quad G_{22} = 0 \\ \kappa_{\alpha_1, -\alpha_1} = \kappa_{\dot{\alpha}_2, -\dot{\alpha}_2} = \kappa_{\dot{\alpha}, -\dot{\alpha}} = 1 \end{aligned} \quad (60)$$

such that the Cartan matrix is given by $A_{ij} = G_{ij}$. The remaining non-vanishing structure coefficients are found to be

$$\begin{aligned} f_{\alpha_1, -\alpha_1}^1 = 1 \quad , \quad f_{\dot{\alpha}_2, -\dot{\alpha}_2}^2 = 1 \\ \dot{\alpha}(H_1) = f_{1, \dot{\alpha}}^{\dot{\alpha}} = 1 \quad , \quad \dot{\alpha}(H_2) = f_{2, \dot{\alpha}}^{\dot{\alpha}} = -1 \\ f_{\dot{\alpha}, -\dot{\alpha}}^1 = f_{\dot{\alpha}, -\dot{\alpha}}^2 = 1 \quad , \quad f_{\pm\dot{\alpha}_2, \mp\dot{\alpha}}^{\mp\alpha_1} = 1 \\ f_{\pm\alpha_1, \pm\dot{\alpha}_2}^{\pm\dot{\alpha}} = \pm 1 \quad , \quad f_{\pm\alpha_1, \mp\dot{\alpha}}^{\mp\dot{\alpha}_2} = \mp 1 \end{aligned} \quad (61)$$

The differential operator realization $\{\tilde{J}_A\}$ is

$$\begin{aligned} \tilde{H}_1(x, \theta, \partial, \Lambda) &= -2x^{\alpha_1} \partial_{\alpha_1} + \theta^{\dot{\alpha}_2} \partial_{\dot{\alpha}_2} - \theta^{\dot{\alpha}} \partial_{\dot{\alpha}} + \Lambda_1 \\ \tilde{H}_2(x, \theta, \partial, \Lambda) &= x^{\alpha_1} \partial_{\alpha_1} + \theta^{\dot{\alpha}} \partial_{\dot{\alpha}} + \Lambda_2 \\ \tilde{E}_{\alpha_1}(x, \theta, \partial) &= \partial_{\alpha_1} - \frac{1}{2} \theta^{\dot{\alpha}_2} \partial_{\dot{\alpha}} \\ \tilde{F}_{\alpha_1}(x, \theta, \partial, \Lambda) &= -x^{\alpha_1} x^{\alpha_1} \partial_{\alpha_1} + \left(\frac{1}{2} x^{\alpha_1} \theta^{\dot{\alpha}_2} - \theta^{\dot{\alpha}} \right) \partial_{\dot{\alpha}_2} \\ &\quad - \frac{1}{2} x^{\alpha_1} \left(\frac{1}{2} x^{\alpha_1} \theta^{\dot{\alpha}_2} + \theta^{\dot{\alpha}} \right) \partial_{\dot{\alpha}} + x^{\alpha_1} \Lambda_1 \\ \tilde{e}_{\dot{\alpha}_2}(x, \theta, \partial) &= \partial_{\dot{\alpha}_2} + \frac{1}{2} x^{\alpha_1} \partial_{\dot{\alpha}} \\ \tilde{f}_{\dot{\alpha}_2}(x, \theta, \partial, \Lambda) &= \left(\frac{1}{2} x^{\alpha_1} \theta^{\dot{\alpha}_2} + \theta^{\dot{\alpha}} \right) \partial_{\alpha_1} + \frac{1}{2} \theta^{\dot{\alpha}_2} \theta^{\dot{\alpha}} \partial_{\dot{\alpha}} + \theta^{\dot{\alpha}_2} \Lambda_2 \\ \tilde{e}_{\dot{\alpha}}(x, \theta, \partial) &= \partial_{\dot{\alpha}} \\ \tilde{f}_{\dot{\alpha}}(x, \theta, \partial, \Lambda) &= -x^{\alpha_1} \left(\frac{1}{2} x^{\alpha_1} \theta^{\dot{\alpha}_2} + \theta^{\dot{\alpha}} \right) \partial_{\alpha_1} - \theta^{\dot{\alpha}_2} \theta^{\dot{\alpha}} \partial_{\dot{\alpha}_2} \\ &\quad + \left(\frac{1}{2} x^{\alpha_1} \theta^{\dot{\alpha}_2} + \theta^{\dot{\alpha}} \right) \Lambda_1 - \left(\frac{1}{2} x^{\alpha_1} \theta^{\dot{\alpha}_2} - \theta^{\dot{\alpha}} \right) \Lambda_2 \end{aligned} \quad (62)$$

The (generalized) Wakimoto free field realization of the associated affine current superalgebra becomes

$$\begin{aligned} H_1(z) &= -2 : \gamma(z) \beta(z) : + : c(z) b(z) : - : C(z) B(z) : + \sqrt{t} \partial \varphi_1(z) \\ H_2(z) &= : \gamma(z) \beta(z) : + : C(z) B(z) : + \sqrt{t} \partial \varphi_2(z) \\ E_{\alpha_1}(z) &= \beta(z) - \frac{1}{2} c(z) B(z) \end{aligned}$$

$$\begin{aligned}
F_{\alpha_1}(z) &= - : \gamma^2(z) \beta(z) : + : \left(\frac{1}{2} \gamma(z) c(z) - C(z) \right) b(z) : \\
&\quad - \frac{1}{2} : \gamma(z) \left(\frac{1}{2} \gamma(z) c(z) + C(z) \right) B(z) : + \sqrt{t} \gamma(z) \partial \varphi_1(z) + (k - 1/2) \partial \gamma(z) \\
e_{\dot{\alpha}_2}(z) &= b(z) + \frac{1}{2} \gamma(z) B(z) \\
f_{\dot{\alpha}_2}(z) &= : \left(\frac{1}{2} \gamma(z) c(z) + C(z) \right) \beta(z) : + \frac{1}{2} c(z) : C(z) B(z) : \\
&\quad + \sqrt{t} c(z) \partial \varphi_2(z) + (k + 1/2) \partial c(z) \\
e_{\dot{\alpha}}(z) &= B(z) \\
f_{\dot{\alpha}}(z) &= - : \gamma(z) \left(\frac{1}{2} \gamma(z) c(z) + C(z) \right) \beta(z) : - : c(z) C(z) b(z) : \\
&\quad + \sqrt{t} \left(\frac{1}{2} \gamma(z) c(z) + C(z) \right) \partial \varphi_1(z) - \sqrt{t} \left(\frac{1}{2} \gamma(z) c(z) - C(z) \right) \partial \varphi_2(z) \\
&\quad + \frac{1}{2} (k - 1) \partial \gamma(z) c(z) - \frac{1}{2} (k + 1) \partial c(z) \gamma(z) + k \partial C(z)
\end{aligned} \tag{63}$$

where we have introduced the simplifying notation

$$\begin{aligned}
\beta(z) &= \beta_{\alpha_1}(z) & , & & \gamma(z) &= \gamma^{\alpha_1}(z) \\
b(z) &= b_{\dot{\alpha}_2}(z) & , & & c(z) &= c^{\dot{\alpha}_2}(z) \\
B(z) &= b_{\dot{\alpha}}(z) & , & & C(z) &= c^{\dot{\alpha}}(z)
\end{aligned} \tag{64}$$

The energy-momentum tensor is

$$\begin{aligned}
T(z) &= : \partial \gamma(z) \beta(z) : + : \partial c(z) b(z) : + \partial C(z) B(z) : \\
&\quad + \frac{1}{2} : \partial \varphi(z) \cdot \partial \varphi(z) : + \frac{1}{\sqrt{t}} \dot{\alpha}_2 \cdot \partial^2 \varphi(z)
\end{aligned} \tag{65}$$

The screening currents of the first kind are

$$\begin{aligned}
s_1(z) &= - \left(\beta(z) + \frac{1}{2} c(z) B(z) \right) : e^{-\varphi_1(z)/\sqrt{t}} : \\
s_2(z) &= - \left(b(z) - \frac{1}{2} \gamma(z) B(z) \right) : e^{-\varphi_2(z)/\sqrt{t}} :
\end{aligned} \tag{66}$$

Proposition 3

The screening current $\tilde{s}_1(w)$ is given by

$$\tilde{s}_1(w) = \left(\beta(w) - \frac{t}{2} c(w) B(w) \right) (\beta(w))^{-t-1} : e^{\sqrt{t} \varphi_1(w)} : \tag{67}$$

and it produces

$$\begin{aligned}
R_{-\alpha_1, \alpha_1} &= t \left(\beta - \frac{1}{2} (t+1) c B \right) \beta^{-t-2} \\
R_{-\dot{\alpha}, \alpha_1} &= t c \beta^{-t-1} \\
R_{-\dot{\alpha}_2, \alpha_1} &= R_{\alpha_1, \alpha_1} = R_{\dot{\alpha}_2, \alpha_1} = R_{\dot{\alpha}, \alpha_1} = R_{i, \alpha_1} = 0 \\
\Delta(\tilde{s}_1) &= 1
\end{aligned} \tag{68}$$

Proof

Also in this case the proof is straightforward. One computes all possible contractions and obtains after some simple rewritings the total derivatives (43) and (68).

□

4 Conclusion

In this paper we have continued the search for screening currents of the second kind in affine current (super-)algebras, and by the explicit construction in the case of $SO(5)$ it has been demonstrated that more complicated structures than previously anticipated [18, 4, 5] appear in the general case. We intend to come back elsewhere with more discussions on the nature of screening currents of the second kind.

Acknowledgement

The author is financially supported by the Danish Natural Science Research Council, contract no. 9700517.

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